# The Explicit Computation of Integration Algorithms and First Integrals for Ordinary Differential Equations With Polynomials Coefficients Using Trees

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**Abstract**

This note is concerned with the explicit symbolic computation of expressions involving differential operators and their actions on functions. The derivation of specialized numerical algorithms, the explicit symbolic computation of integrals of motion, and the explicit computation of normal forms for nonlinear systems all require such computations. More precisely, if $R = k[x_1, \ldots, x_N]$, where $k = \mathbf{R}$ or $\mathbf{C}$, $F$ denotes a differential operator with coefficients from $R$, and $g \in R$, we describe data structures and algorithms for efficiently computing $F \cdot g$. The basic idea is to impose a multiplicative structure on the vector space with basis the set of finite rooted trees and whose nodes are labeled with the coefficients of the differential operators. Cancellations of two trees with $r + 1$ nodes translates into cancellation of $O(N^r)$ expressions involving the coefficient functions and their derivatives.

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## Status

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